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TURBULENT SCALAR TRANSPORT CORRELATION BEHIND A LINE HEAT SOURCE IN A UNIFORM SHEAR FLOW

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Measured profiles of third order scalar-velocity correlations, which appear in the governing equations of Reynolds stress and turbulent heat flux, are presented for the turbulent diffusion field in a homogeneous shear flow. Assessment of previous models for 3rd order moments (Launder's model, Lumley's model) using the experimental values of lower order moments and time scales reveal that the accuracies of predictions are different from quantity to quantity. In order to predict the 3rd order moments more accurately, a new composite time scale for the simple gradient transport model is proposed.

Key Words: Reynolds Stress Closure Model, Time Scale, Third Order Scalar-Velocity Correlations

NOMENCLATURE

C_s, b	: Model constants
d	: Diameter of line heat source
L	: Length scale
R	: Time scale ratio
u, v, w	: Fluctuating velocity components in axial, vertical and lateral directions, respectively
x	: Axial distance from the line heat source
y	: Vertical distance from the line heat source
θ	: Fluctuating temperature component
τ_u	: Dynamic time scale
τ_θ	: Thermal time scale
τ_3	: Time scale for the simple gradient transport model
ε	: The dissipation rate of kinetic energy
ε_θ	: The dissipation rate of thermal fluctuation
u	: Velocity scale

Subscripts

$m(ax)$	: Maximum

Superscript

$'$	: rms value
$\overline{(\ )}$	: Conventional time average

1. INTRODUCTION

Since the establishment of the Reynolds-stress closure model for isothermal turbulent flows in the mid 1970's, turbulent transports of scalar quantities have attracted much attention from theoretical and experimental investigators. Like the modeling procedure for the isothermal turbulent flows as have been well documented by Reynolds(1978), a systematic way to develop the second-order closure model for the non-isothermal turbulent flow is to consider on turbulence mechanism at a time in the order of increasing complexity; for example, in a sequence of determining decay rate constant of scalar variance, return-to-isotropy of scalar flux vector and then third-order scalar transports, etc. At each stage of modeling, a simplest experiment which exhibits only the mechanism under consideration must be available to calibrate the model constant(or term). Current gradient transport models for the triple velocity-scalar correlations contain time scales like the decay rate constant for grid-generated turbulence. Typical time scale in the majority of turbulence models is the dynamic time scale(or eddy turn-over time). Other one is the composite time scales proposed by Zeman and Lumley(1979) to give better predictions in a buoyancy driven mixed layer. But both have not been sufficiently tested because of the lack of comparable experimental data.

The first systematic experimental approach to the problem of scalar turbulence has been done by Warhaft and Lumley(1978) who studied the decaying rate of the temperature variance $\overline{\theta^2}$ in approximately isotropic grid-generated turbulence. Based on these data, Newman, Launder and Lumley(1981) proposed a model for the decay rate "constant" as a function of the mechanical/thermal time scale ratio $R(=(\overline{\theta^2}/\varepsilon_\theta)/(\overline{q^2}/\varepsilon))$, local Reynolds number and anisotropy of turbulence. The effect of uniform strain and the effect of temperature gradient are investigated for the formulation of a return to isotropy model "constant" appearing in a scalar flux equation $\overline{u_j\theta}$. (Tavoularis and Corrsin, 1980, Sirivat and Lumley, 1983, Shih and Lumley, 1986, Budwig et al, 1985)

Finally, for completion of the closure problem of the thermal turbulence at the second-order level, experimental informations on the triple products between velocity and temperature, or the third-order scalar transport terms, for example $\overline{u_iu_ju_k}$, $\overline{u_iu_j\theta}$ and $\overline{u_j\theta^2}$, have been wanted. One such experimental endeavor has been done by Fabris(1983) who obtained data of all conditional scalar transport terms in a two dimensional turbulent wake behind a heated circular cylinder bar.

Raupach and Legg(1983) obtained quite complete turbulence correlations at first, second and third-orders behind an elevated line heat source in a turbulent boundary layer. Dekeyser and Launder(1983) measured all the scalar trans-

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port terms in a heated asymmetric two dimensional turbulent jet. In addition to the presentation of the data, they carried out numerical tests of currently available gradient-diffusion type models for the scalar-velocity triple products, and they suggested that an extensive theoretical model of Lumley(1978) for the scalar transport terms would be worth examining to see if such an extensive and highly complicated model would give better prediction.

The objectives of the present study are to provide complete statistical data for the third order scalar transport terms and to develop a new simple gradient transport model for the third-order scalar transports with reference to our experimental data.

In addition, currently available turbulence models for the triple correlations are compared with our proposed simple gradient transport model with a composite time scale.

2. TURBULENCE MODELS FOR THIRD-ORDER SCALAR TRANSPORT TERMS

A number of studies, theoretical and experimental in nature, have been reported on the modeling of the turbulent third order scalar transport terms. Among these, Dekeyser and Launder(1983) extensively tested closure models for the triple moments using their experimental data on an asymmetric heated jet. The dispersion of scalar contaminants by an elevated line heat source in a turbulent boundary layer was measured by Raupach and Legg(1983), who compared their data of triple temperature-velocity correlations with model predictions. Both studies revealed that the current third order transport models incorporating the dynamic time scale only have limited accuracy in predicting the moments. The models are in reasonable agreement with the purely dynamic correlating terms, $\overline{u^2v}$, $\overline{uv^2}$ and $\overline{v^3}$, but significant discrepancies exist for temperature-velocity correlations, $\overline{uv\theta}$, $\overline{v^2\theta}$ and $\overline{v\theta^2}$.

This inaccuracy stems from neglecting non-negligible contributions by the thermal fluctuations in spatial transport of the second order quantities. Zeman and Lumley(1979) have shown that the thermal time scale τ_θ has a significant role in the evolution process of turbulent flows in the buoyancy-driven mixed convection. They have also proposed to use a composite time scale as a weighted geometric mean between the dynamic time scale and the thermal time scale for the third-order transport terms. One of the purpose in this section is to propose an appropriate time scale in representing the spatial transports of $\overline{uv}$, $\overline{u\theta}$ and $\overline{\theta^2}$ in nonisothermal turbulent flows with negligible buoyancy effect. Such transport terms have usually been approximated by simple gradient transport models. Specifically, for boundary-layer-type flows, they are as follows:

$$\overline{uv^2}=-\tau_3\left(2\overline{v^2}\frac{\partial\overline{uv}}{\partial y}+\overline{uv}\frac{\partial\overline{v^2}}{\partial y}\right)$$
$$\overline{uv\theta}=-\tau_3\left(\overline{uv}\frac{\partial\overline{v\theta}}{\partial y}+\overline{v^2}\frac{\partial\overline{u\theta}}{\partial y}\right)$$
$$\overline{v^2\theta}=-2\tau_3\overline{v^2}\frac{\partial\overline{v\theta}}{\partial y}$$
$$\overline{v\theta^2}=-\tau_3\overline{v^2}\frac{\partial\overline{\theta^2}}{\partial y}$$

where τ_3 is an appropriate time scale for the third-order moments. It has often been asserted that the time scale τ_3 may be subsituted by the dynamic time scale τ_u. This is possible if one assumes that the thermal time scale τ_θ is almost proportional to τ_u and thus $\tau_3=0.1\frac{q^2}{2\varepsilon}$ with a different time scale constant. However, since the time scale ratio $R=\tau_\theta/\tau_u$ has been proved to vary in the range of $0.5<R<1.5$, such an assumption is not adequate.

Under further assumptions of weak inhomogeneity in temperature and negligible buoyancy effect, Lumley(1978) presented a closed set of linear equations for $\overline{u_j\theta^2}$, $\overline{u_iu_j\theta}$ and $\overline{u_iu_ju_k}$ in a tensorial form. These two models are well supported by the basic theory of turbulence. For example, Lumley(1978) has proposed to solve the following system of linear equations for the third-order scalar transports, $\overline{u_iu_ju_k}$, $\overline{u_j\theta^2}$ and $\overline{u_iu_j\theta}$:

$$\overline{\theta^2}_{,k}\overline{u_ju_k}+2(\overline{u_j\theta})_{,k}\overline{u_k\theta}=-2\frac{\varepsilon_\theta}{\overline{\theta^2}}\left(1+c\frac{\varepsilon}{\theta\overline{q^2}}\frac{\overline{\theta^2}}{\varepsilon_\theta}\overline{u_j\theta^2}\right)$$
$$(u_1\theta)_{,k}\overline{u_ku_j}+(u_j\theta)_{,k}\overline{u_ku_i}+(u_iu_j)_{,k}\overline{u_k\theta}$$
$$=-\left[c_1\frac{\varepsilon}{q^2}\left(\overline{u_iu_j\theta}-\frac{1}{3}\overline{q^2\theta}\delta_{ij}\right)+\frac{2}{3}\frac{\varepsilon}{q^2}\overline{q^2\theta}\delta_{ij}\right.$$
$$\left.+2c\theta\frac{\varepsilon}{q^2}\overline{u_iu_j\theta}\right]$$
$$(\overline{u_iu_j})_{,p}u_ku_p+(u_iu_k)_{,p}u_ju_p+(u_ju_k)_{,p}u_iu_p$$
$$=-3c_1\left(\frac{\varepsilon}{q^2}\right)\left[\overline{u_iu_ju_k}-\frac{1}{9}\left(\delta_{ij}\overline{q^2u_k}+\delta_{ik}\overline{q^2u_j}\right.\right.$$
$$\left.\left.+\delta_{jk}\overline{q^2u_i}\right)\right]-\frac{2}{3}\frac{\varepsilon}{q^2}\left(\delta_{ij}\overline{q^2u_k}+\delta_{ik}\overline{q^2u_j}+\delta_{ik}\overline{q^2u_i}\right)$$

However, resulting explicit forms for the transport terms under consideration are too complicated and laborious to use in the computation of the non-isothermal turbulent flow at the second order level.

In the present study, a rather simple weighted algebraic mean between τ_u and τ_θ is proposed to be used as a simple composite time scale in conjunction with conventional gradient transport model given above (Chung and Kyong, 1986);

$$\tau_3=C_s(\tau u+b\tau_\theta)=C_s\tau u(1+bR)$$

Here, we put the constant C_s to a value which has been used customarily in isothermal turbulence, $C_s=0.055$. The constant b is assumed to depend on the power of the temperature fluctuations θ in the moments: namely, $b=0$ for $\overline{u^2v}$, $\overline{uv^2}$ and $\overline{v^3}$, $b=1$ for $\overline{uv\theta}$ and $\overline{v^2\theta}$, and $b=2$ for $\overline{v\theta^2}$. This model reflects the fact that a lager contribution come from the thermal time scale for higher powers of θ in the triple moments.

3. RESULT AND DISCUSSION

Model predictions by Launder's simple gradient transport model(1978) and Lumley's model quoted in the previous chapter and our new gradient transport model with the composite time scale are compared in the following discussions together with the presentation of experimental data. The experimental data are obtained by the same facilities as explained in Kyong and Chung(1987). Measured third order moments show larger scattering than the second order moments presented in Kyong and Chung(1987), but the scatterings of experimental data are within 10% of the average values.

All predictions are computed with the directly measured

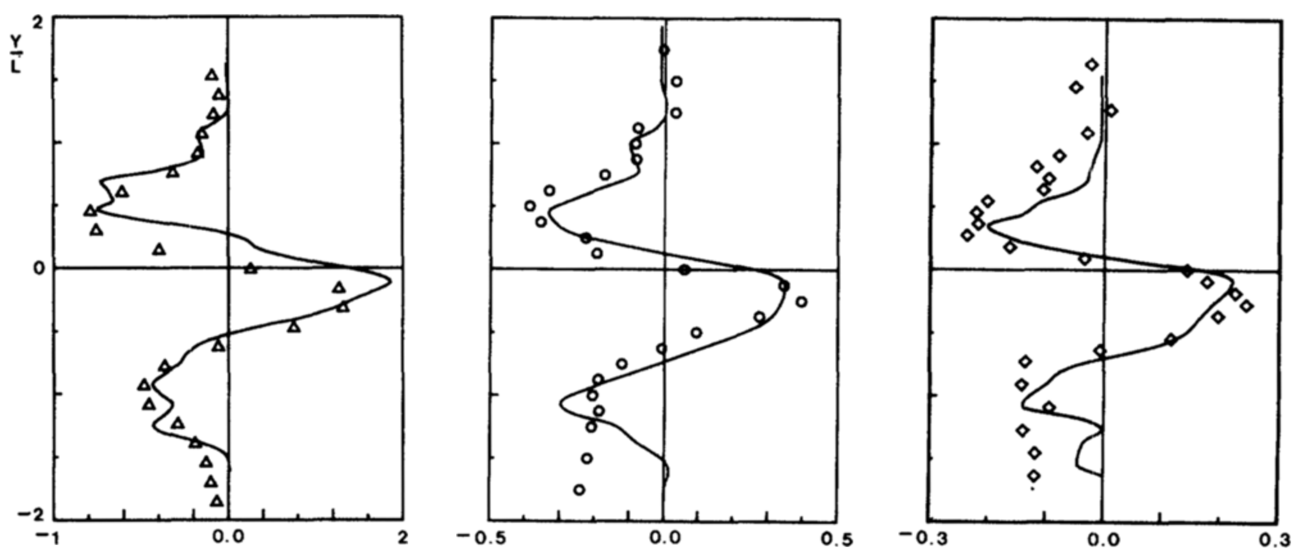

Fig. 1 Comparison of model predictions with data of $\overline{uv^2}/u^3$ where $u^2 = \overline{u_i u_i}/3$: Predictions ; ——, Present model ; -----, Lumley ; — —, Launder : △, x/d = 900 ; ○, x/d = 1650 ; ◇, x/d = 2390

values. In the following figures, the geometric symbols represent our data, the solid line is the predicted profile by our gradient-diffusion model, the dotted line by Lumley's model and the dashed line by the simple gradient transport model. Fig. 1 shows a comparison of the predictions with the data of $\overline{uv^2}$, the streamwise transport of $\overline{v^2}$ and vertical transport of $\overline{uv}$. (Here, Lumley's model was not tested due to the lack of data on transverse velocity fluctuations w) The profiles are generally similar to those of a heated plane wake. (Fabris, 1983) The vertical position of the positive peak in lower part coincides with the maximum velocity wake. This is in contrast with the heated plane jet (Dekeyser and Launder, 1983) where $\overline{uv^2}$ is negative in the central part and becomes positive in the outer free mixing zone. The simple gradient transport model predicts $\overline{uv^2}$ fairly well. The abnormal variations of the predicted profiles at both peaks at x/d = 900 of Fig. 1 are due to experimental uncertainty in the second-order moments.

Figure 2 shows profiles of $\overline{uv\theta}$, the streamwise transport of $\overline{v\theta}$. The profile is similar to that in the dispersion measurement from a line heat source in a turbulent boundary layer(Raupach and Legg, 1983) excluding the region near the wall layer. In the plane thermal wake, $\overline{uv\theta}$ vanishes at the wake center and changes sign from positive to negative along the outward direction. But, in our case and in the turbulent boundary layer, $\overline{uv\theta}$ does not vanish at the thermal jet center. Predictions are fairly reasonable except for Lumley's model which underpredicts $\overline{uv\theta}$ in the whole region by a factor of about 2. In addition, Lumley's model particularly fails to predict $\overline{ve\theta}$ at x/d = 900.

Figure 3 represents the streamwise transport of $\overline{u\theta}$, i.e., $\overline{u^2\theta}$ profiles. The profiles are almost completely positive in the upper and lower parts which differ from the heated jet where it has smaller negative values in the central region. All

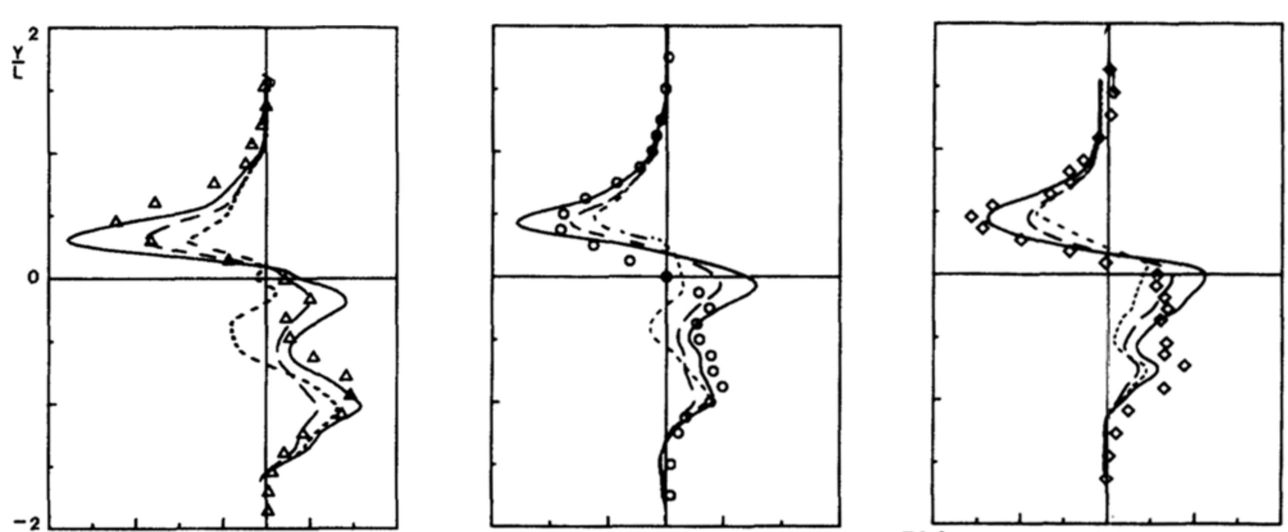

Fig. 2 Comparison of model predictions with data of $\overline{uv\theta}/u^2 T_m$ where $u^2 = \overline{u_i u_i}/3$: Predictions ; ——, Present model ; -----, Lumley ; — —, Launder : △, x/d = 900 ; ○, x/d = 1650 ; ◇, x/d = 2390

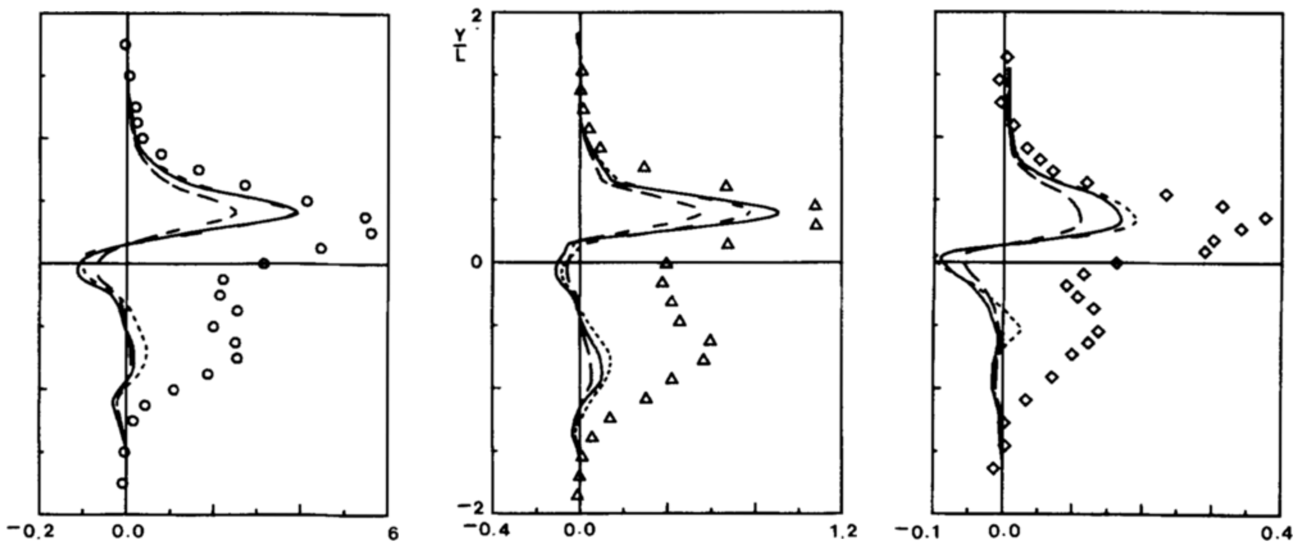

Fig. 3 Comparison of model predictions with data of $\overline{u^2\theta}/u^2 T_m$ where $u^2 = \overline{u_i u_j}/3$: Predictions ; ——, Present model ; -----, Lumley ; — —, Launder : △, $x/d = 900$; ○, $x/d = 1650$; ◇, $x/d = 2390$

models underpredict the $\overline{u^2\theta}$ experimental values. This may imply that the timescale for the streamwise transport should be different from that for vertical transport. In Fig. 4, profiles of $\overline{u\theta^2}$, the streamwise transport of $\overline{\theta^2}$, are presented. $\overline{u\theta^2}$ remains negative with two peaks in both sides. In the heated plane wake, the $\overline{u\theta^2}$ which is slightly positive near the center becomes highly negative in the outer free mixing region. However, in the heated plane jet, the $\overline{u\theta^2}$ profile is highly negative in the entire upper part of the jet but has a peak positive value in a narrow region of the lower part. All models predict only 20% of the $\overline{u\theta^2}$ level in the whole field. Again, this implies that the streamwise transport of $\overline{\theta^2}$ must have a different timescale from other vertical transport.

The vertical transport of the vertical heat flux, $\overline{v^2\theta}$ is represented in Fig. 5. The vertical gradient of $\overline{v^2\theta}$ is the turbulent diffusion term in the boundary-layer type governing equation of $\overline{v\theta}$. Two zero crossings are located near the peaks of $\overline{v\theta}$ and the negative peak coincide with the zero crossing of $\overline{v\theta}$.(Kyong and Chung, 1987) This means that the diffusion of $\overline{v\theta}$ by turbulent vertical fluctuation is most severe at the peaks of $\overline{v^2\theta}$ since the maximum slopes are near those points in opposite sign, which result in the compression of two peaks. And the diffusion of $\overline{v\theta}$ by v is negligible at the zero crossing of $\overline{v^2\theta}$. The profiles are similar to the heated plane wake but with downward shift of the profile. The same trend can also be seen in boundary layer or in heated jet. The predictions by the simple gradient-transport model of Launder's and Lumley's are fairly good, but our model overpredicts the profile by about 30%. Finally, Fig. 6 show the profiles of $\overline{v\theta^2}$, the vertical transport of the temperature variance, appearing as the diffusion term in the governing equation of $\overline{\theta^2}$. A remarkable feature is that all the profiles have nearly flat region at the center corresponding to the profiles of temperature variances. This imply that the diffu-

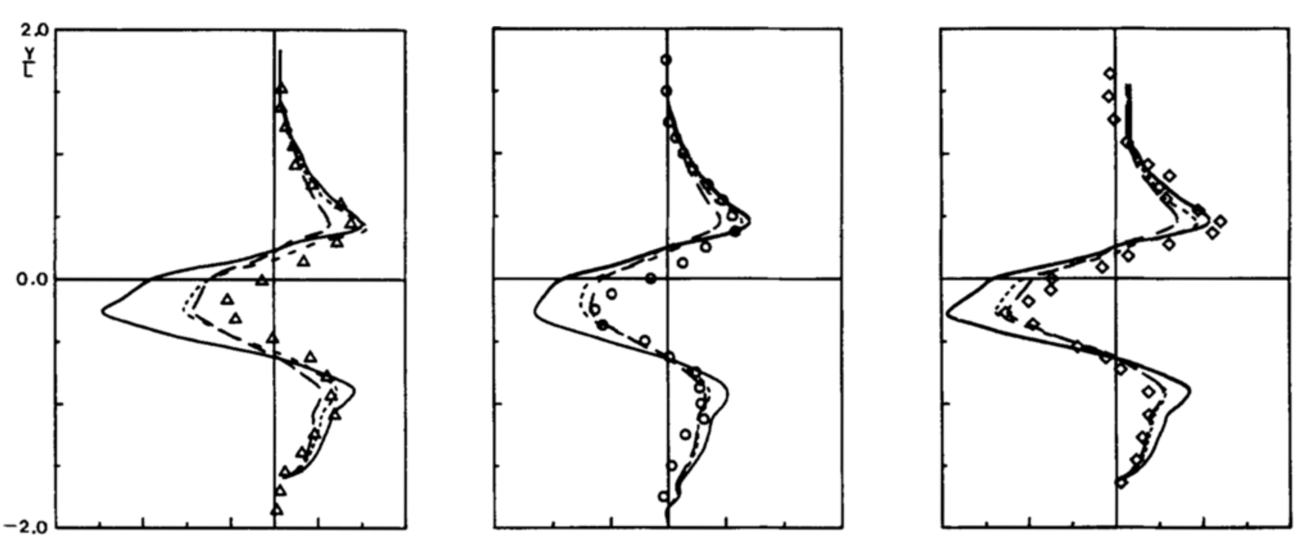

Fig. 4 Comparison of model predictions with data of $\overline{u^2\theta}/u^2 T_m$ where $u^2 = \overline{u_i u_j}/3$: Predictions ; ——, Present model ; -----, Lumley ; — —, Launder : △, $x/d = 900$; ○, $x/d = 1650$; ◇, $x/d = 2390$

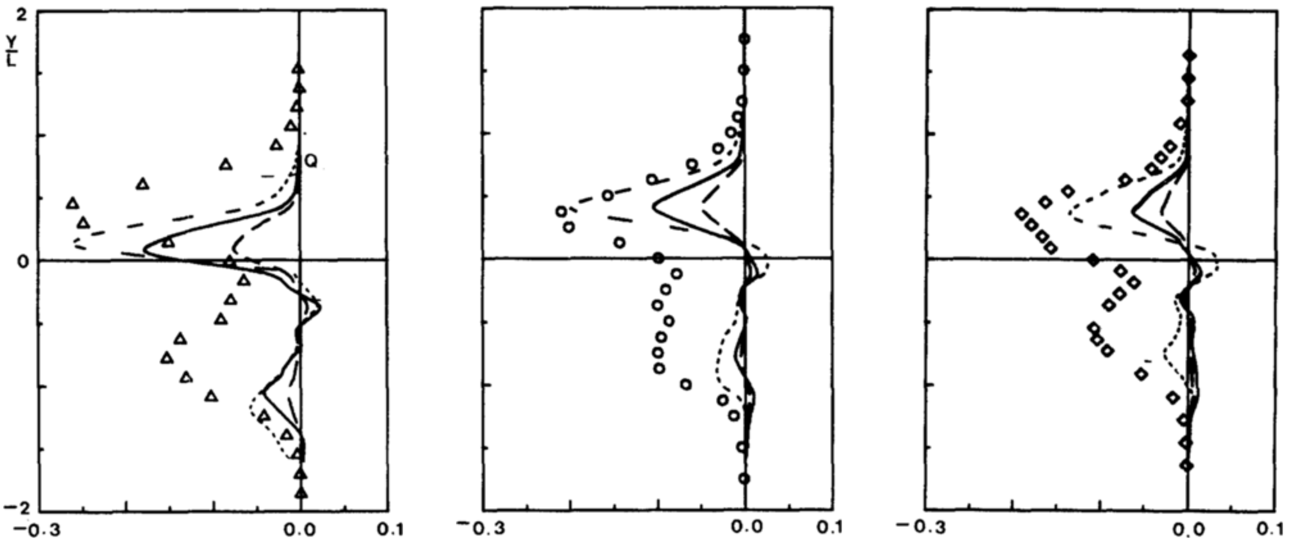

Fig. 5 Comparison of model predictions with data of $\overline{u\theta^2}/uT_m^2$ where $u^2 = \overline{u_iu_j}/3$: Predictions; ——, Present model; -----, Lumley; ——, Launder: △, x/d = 900; ○, x/d = 1650; ◇, x/d = 2390

sions of $\overline{\theta^2}$ by v are negligible at those regions. The two peaks coincide with the maximum slope point in θ-profiles as can be seen in Launder's model, where the $\overline{v\theta^2}$ is denoted by the simple gradient of $\overline{\theta^2}$ in the boundary-layer-type flow. This also means that the diffusions of $\overline{\theta^2}$ by v are negligible at the points of maximum gradient of $\overline{\theta^2}$ profiles. The profile shows zero value at the centerline of the thermal jet. This is consistent with the profile in the heated plane wake, the heated jet and thermal jet in the turbulent boundary layer. Our model and Lumley's model yield very good predictions on the $\overline{v\theta^2}$ profile, whereas the Launder's model underpredicts the profiles.

Considering the overall performance of the three models for all the triple moments in this section, it may be concluded that our model which employs a composite time scale, yields the best predictions with no additional cost.

4. CONCLUSIONS

As a result of the model predictions, all models considered here provide correct trend of the profiles but with different level of prediction accuracy. It was found that the simple gradient-transport model with a composite timescale proposed in this study yields better numerical values in comparison with more extensive theoretical models. An important observation is that all the streamwise transports of second-order moment are badly underpredicted compared with the vertical transports of the same moments. This strongly suggests that the streamwise transport must have timescales different from those for the vertical or cross-stream transport of the second-order moments.

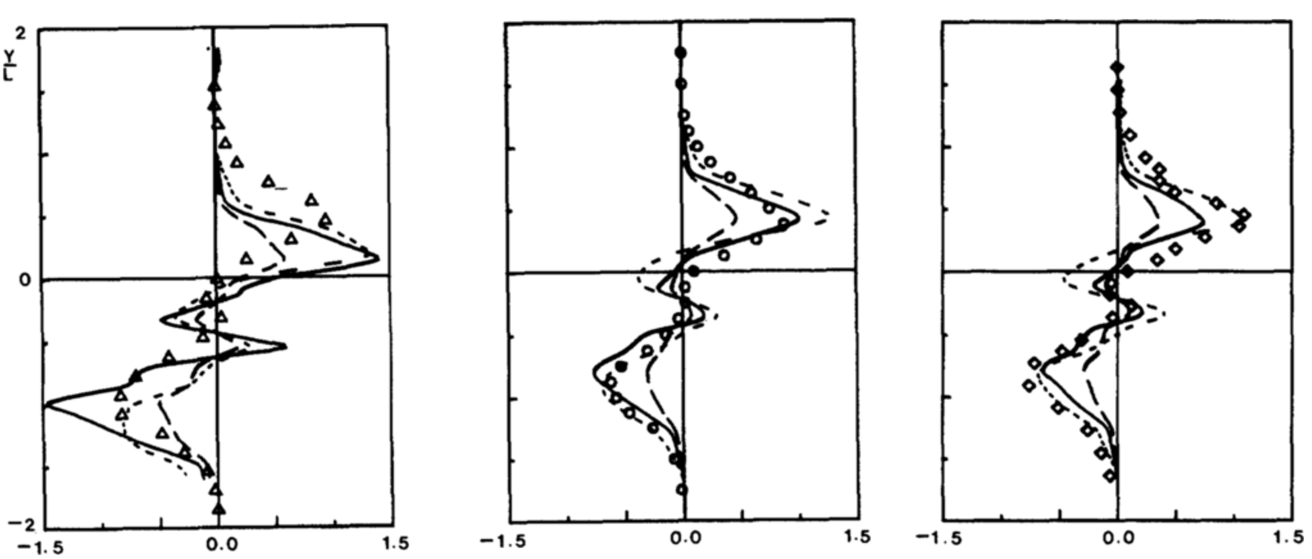

Fig. 6 Comparison of model predictions with data of $\overline{u\theta^2}/uT_m^2$ where $u^2 = \overline{u_iu_j}/3$: Predictions; ——, Present model; -----, Lumley; — —, Launder: △, x/d = 900; ○, x/d = 1650; ◇, x/d = 2390

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REFERENCES

Budwig, R., Tavoularis, S. and Corrsin, S., 1985, "Temperature Fluctuations and Heat Flux in Grid-generated Isotropic Turbulence with Streamwise and Transverse Mean Temperature Gradients", J. Fluid Mech., Vol. 153, pp. 441~460.

Chung, M.K. and Kyong, N.H, 1986, "A Simple Composite Time Scale for Third Order Scalar Transport", Phys. of Fluid, Vol. 29, No. 12, pp. 3914~3916.

Dekeyser, I. and Launder, B.E., 1983, "A Comparison of Triple Moment Temperature-Velocity Correlations in the Asymmetric Heated Jet with Alternative Closure", Turbulent Shear Flow 4, pp. 102~120, Springer-Verlag.

Fabris, G., 1983, "Third Order Conditional Transport Correlations in the Two-dimensional Turbulent Wake", Phys. of Fluid, Vol. 26, No. 2, pp. 422~427.

Kyong, N.H. and Chung, M.K., "Measurement of Turbulent Diffusion Field Behind a Line Heat Source in a Homogeneous Shear Flow", J. of KSME, Vol. 1, No. 1, pp. 24~30.

Launder, B.E., 1978, "Heat and Mass Transport: Turbulence", Ed. by P. Bradshaw, Springer-Verlag.

Lumley, J.L., 1978, "Computational Modeling of Turbulent Flows", Advances in Applied Mechanics, Vol. 18, pp. 123~176, Academic Press.

Newman, G.R., Launder, B.E. and Lumley, J. L., 1981, "Modeling the Behavior of Homogeneous Scalar Turbulence", J. Fluid Mech., Vol. 111, pp. 217~232.

Raupach, M.R. and Legg, B.J., 1983, "Turbulent Dispersion From Elevated Line Source", J. Fluid Mech., Vol. 136, pp. 111~137.

Reynolds, W.C. and Cebeci, T., 1978, "Calculation of Turbulent Flows: Turbulence", Ed. P. Bradshaw, Springer-Verlag.

Shih, T. and Lumley, J.L., 1986, "Influence of Timescale Ratio on Scalar Flux Relaxation: Modeling Sirivat & Warhaft's Homogeneous Passive Scalar Fluctuations, J: Fluid Mech., Vol. 162, pp. 211~222.

Sirivat, A. and Warhaft, Z., 1983, "The Effect of a Passive Cross-stream Temperature Gradient on the Evolution of Temperature Variance and Heat Flux in a Grid Turbulence", J. Fluid Mech., Vol. 128, pp. 323~346.

Tavoularis, S. and Corrsin, S., 1980, "Experiments in Nearly Homogeneous Turbulent Shear Flow with a Uniform Mean Temperature Gradient", J. Fluid Mech., Vol. 104, pp. 317~347.

Warhaft, Z. and Lumley, J.L., 1978, "An Experimental Study of the Decay of Temperature Fluctuations in Grid-Generated Turbulence", J. Fluid Mech., Vol. 88, pp. 659~684.

Zeman, O. and Lumley, J.L., 1979, "Buoyancy Effects in Entraining Turbulent Boundary Layer: A Second-Order Closure Study", Turbulent Shear Flow I, pp. 280~294, Springer-Verlag.